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STABILITY OF THE LAMINAR BOUNDARY LAYER OVER A DEFORMABLE DIAPHRAGM TYPE SURFACE

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ABSTRACT. The stability of the Blasius flow over a diaphragm-type surface, the physical characteristics of which are constant along the length, is examined.

Attempts have been made to provide a theoretical explanation of the effect boundary deformation has on the position of the point in the boundary layer at which stability is lost. These attempts are associated with Kramer's successful experiments [1, 2] in sheathing models with flexible coatings. A. I. Korotkin [3] examined the stability of a plane laminar boundary layer over an elastic surface on the assumption that there is a linear connection between pressure disturbance and normal surface deformation. Benjamin [4] and Landahl [5] investigated the stability of a laminar boundary layer over a diaphragm-type surface on the assumption that the physical characteristics of the surface depend on the wavelength of the disturbing flow. /52*

The stability of the Blasius flow over a diaphragm-type surface, the physical characteristics of which are constant along the length, will be examined in what follows.

We will take it that when there are no disturbances the surface of the plate coincides with the half-plane $x \geqslant 0$, $y = 0$ (Figure 1). Let us suppose that certain disturbances have taken place in the flow at a predetermined moment. Let us investigate the stability of the flow with respect to these disturbances.

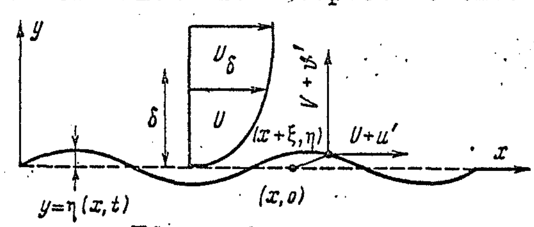

Figure 1

* Numbers in the margin indicate pagination in the foreign text.

Let U, V ($V \ll U$) be the components of the velocity of the Blasius flow along the x and y axes, respectively, p be the pressure, ν be the kinematic modulus of viscosity, and ρ be the fluid density. We will take it that the velocities of the disturbances, u', v', and the pressure disturbance, p', are small in the sense that terms that are quadratic with respect to the disturbances can be ignored. Let us introduce the stream function, ψ', for the disturbing flow in the form

$$\psi' = \varphi(y) \exp[i\,\alpha(x-ct)] \tag{1}$$

the while assuming that the real part of equation (1) is taken. The wave number, α, is a real magnitude, linked with the wavelength of the disturbing flow by the relationship $\alpha = 2\pi/\lambda$. The phase velocity $c = c_r + ic_i$ is a complex magnitude. The sign of the imaginary part, c_i, tells whether the disturbance will increase ($c_i > 0$), or be damped ($c_i < 0$). Dimensionless magnitudes are used in equation (1), as well as in what follows. The velocity U_δ at the outer limit of the boundary layer is taken as the velocity scale, and the thickness of the boundary layer

$$\delta = 6\sqrt{\frac{\nu x}{U_\delta}}$$

is taken as the length scale.

The neutral curve $c_i = 0$, separating the region of rising disturbances from the region of damping disturbances, is of particular interest. The Reynolds number for loss of stability is determined by the shape of this curve. The neutral stability curve is constructed from the solution of the Orr-Sommerfield /53
equation for the amplitude φ of the stream function for a disturbing flow [6]

$$(U-c)(\varphi''-\alpha^2\varphi) - U''\varphi = \frac{1}{\alpha i R}(\varphi^{IV} - 2\alpha^2\varphi'' + \alpha^4\varphi) \tag{2}$$

where

$$R = \frac{\delta U_\delta}{\nu}, \qquad U = 2y - 5y^4 + 6y^5 - 2y^6$$

The boundary conditions for equation (2) express the conditions for disturbance damping at infinity and the adhesion conditions. The conditions at infinity are in the form [6]

$$\varphi' + \alpha\varphi = 0, \quad |\varphi| < \infty, \tag{3}$$

The adhesion conditions express the equality of the velocity of a surface element and a liquid particle adjacent to the surface (Figure 1)

$$\begin{aligned}\frac{\partial \xi (x, t)}{\partial t} &= U (x + \xi, \eta) + u' (x + \xi, \eta)\\ \frac{\partial \eta (x, t)}{\partial t} &= V (x + \xi, \eta) + v' (x + \xi, \eta).\end{aligned} \tag{4}$$

Let us put

$$\xi (x, t) = \xi_1 e^{i\alpha (x-ct)}, \qquad \eta (x, t) = \eta_1 e^{i\alpha (x-ct)} \tag{5}$$

Substituting the equality at (5) in (4), expanding the right sides of the latter in a Taylor series, and taking the smallness of the deformations, and the velocity V, into consideration, we obtain

$$\xi_1 = -\frac{i}{\alpha c}\left[\frac{U'(0)\varphi(0)}{c} + \varphi'(0)\right], \qquad \eta_1 = \frac{\varphi(0)}{c} \tag{6}$$

It will be convenient, in subsequent computations, to introduce the normal, Y_o, and the tangential, X_o, yielding of the flow with respect to the traveling wave. The normal (tangential) yielding is determined with sign correctness by the ratio of the normal (tangential) velocity to the pressure disturbance, $p' = p_1 \exp [i\alpha (x - ct)]$, that is

$$\begin{aligned}Y_0 &= -\frac{V (x + \xi, \eta) + v' (x + \xi, \eta)}{p' (x + \xi, \eta)}\\ X_0 &= \frac{U (x + \xi, \eta) + u' (x + \xi, \eta)}{p' (x + \xi, \eta)}\end{aligned}$$

which can be written with first order of infinitesimals correctness as

$$Y_0 = \frac{i\alpha\varphi (0)}{p_1 (0)}, \qquad X_0 = \frac{1}{p_1(0)}\left[\frac{\varphi (0)}{c} U'(0) + \varphi'(0)\right] \tag{7}$$

The amplitude p_1 of the pressure disturbance can be found from the linearized equations for the motion of a viscous fluid in projections on the x and y axes, respectively

$$p_1 = \frac{1}{i\alpha R}[\varphi'''(0) - \alpha^2\varphi'(0)] + c\varphi'(0) + U'(0)\varphi(0) \tag{8}$$

or

$$p_1 = -\int_0^\infty \left[\frac{\alpha}{iR}(\varphi'' - \alpha^2\varphi) - \alpha^2 (U - c)\varphi\right] dy \qquad (9)$$

The identity of equations (8) and (9) follows from equation (2).

Introduced in similar fashion are the tangential, Y_{12}, and normal, Y_{11}, yieldings of a deformable surface with respect to the traveling wave

$$Y_{12} = \frac{1}{p'}\frac{\partial \xi}{\partial t}, \qquad Y_{11} = -\frac{1}{p'}\frac{\partial \eta}{\partial t}$$

and can be written with first order of infinitesimals correctness in the form /54

$$Y_{12} = -\frac{i\alpha c\xi_1}{p_1}, \qquad Y_{11} = \frac{i\alpha c\eta_1}{p_1} \qquad (10)$$

The equality

$$Y_0 = Y_{11},\ X_0 = Y_{12} \qquad (11)$$

yields the boundary conditions on the deformable surface.

The computations show that the tangential yielding has little effect on the position of the point at which stability is lost, and it can be taken as equal to zero.

Let us, in order to determine the normal yielding, Y_{11}, which is dependent on η_1, consider the motion of a diaphragm element (Figure 2).

$$m\frac{\partial^2\eta}{\partial t^2} = -p' - k\eta + T\frac{\partial^2\eta}{\partial x^2} - d\frac{\partial\eta}{\partial t}$$
$$m = \frac{m_*}{\rho\delta} = \frac{k_m}{R}, \qquad p' = \frac{p_*'}{\rho U_\delta^2}, \qquad k = \frac{k_*\delta}{\rho U_\delta^2} \qquad (12)$$
$$T = \frac{T_*}{\delta\rho U_\delta^2}, \qquad d = \frac{d_*}{\rho U_\delta}$$

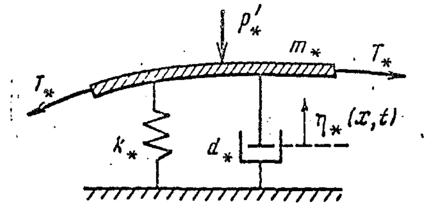

Figure 2

Here m_* is the mass of a unit of diaphragm area, T_* is the surface tension occurring per unit width of the diaphragm, and k_* is the stiffness factor. The asterisks denote dimensional magnitudes.

Let us find the η_1/p_1 ratio from equation (12), with equation (5) taken into consideration. When this ratio is substituted in equation (10) we obtain

$$Y_{11} = -\frac{i\alpha c}{m\alpha^2 (c_0^2 - c^2 - cid/m\alpha)} \qquad (13)$$

where

$$c_0^2 = c_{0m}^2 + \frac{\omega_0^2}{\alpha^2}, \quad c_{0m}^2 = \frac{T}{m}, \quad \omega_0^2 = \frac{k}{m} = k_\omega^2 R^2$$

An approximate solution of equation (2) can be given in the form

$$\varphi = \Phi + A\varphi_3 \qquad (14)$$

In this equation Φ is the "nonviscous" solution, satisfying the equation

$$(U - c)(\Phi'' - \alpha^2\Phi) - U''\Phi = 0 \qquad (15)$$

and φ_3 is the approximate "viscous" solution of equation (2) satisfying equation [6]

$$\frac{d^4\varphi_3}{d\eta_2^4} - i\eta_2 \frac{d^2\varphi_3}{d\eta_2^2} = 0, \quad \eta_2 = \frac{y - y_c}{\varepsilon}, \quad \varepsilon = (\alpha R U_c')^{-1/3} \qquad (16)$$

Here Y_c is the value of y for which U = c.

The solutions of Φ and φ_3 satisfy the boundary conditions at (3). The boundary conditions at (11), and the condition of nontriviality of the solution lead to the characteristic equation linking the magnitudes α, c, R with the parameters of the deformable surface. Before writing this equation, let us simplify the expression for the pressure amplitude, p_1, contained in (11), ignoring the small magnitude terms. In accordance with equations (8) and (14), we can write

$$p_1 = \frac{\Phi'''(0) - \alpha^2\Phi'(0)}{i\alpha R} + A\frac{\varphi_3'''(0) - \alpha^2\varphi_3'(0)}{i\alpha R} + c\varphi'(0) + U^\varphi(0)\varphi(0) \quad (17)$$

The first term in the right side of equation (17) can be ignored because the change in the nonviscous solution is slow. This term is exactly equal to zero in the case of the Blasius flow, as follows from equation (15) after differentiation with respect to y. The sum of the third and fourth terms in the right side of equation (17), in accordance with equations (6) and (10), equals $cY_{12}p_1$, and it too can be ignored. In order to make further simplifications in equation (17), let us find $\varphi_3'''(0)$ from equation (16) by term integration with respect to y /55

$$\varphi_3'''(0) = -i\alpha R y_c U_c' \varphi_3'(0)\left[1 + \frac{\varphi_3(0)}{y_c\varphi_3'(0)}\right] \quad (18)$$

Equation (18) yields $|\varphi_3'''(0)| \gg |\varphi_3'(0)|$. Therefore, taking the fact that $U_c' \approx U'(0)$, $y_c U_c' \approx c$, and using equations (7) and (11), our final finding is

$$p_1(0) = U'(0)\Phi(0) + c\Phi'(0) \quad (19)$$

An identical expression can also be obtained by the transformation of equation (9), and it will be correct to within the $R^{-1/3}$ terms. The arguments cited above confirm Landahl's assertion [5] that a linearized equation of motion in the projection on the y axis yields a more accurate expression for pressure disturbance than does a linearized equation of motion in the projection on the x axis.

Using the expression obtained for p_1, we can write the characteristic equation in the form

$$\{Y_{11}[U'(0)\Phi(0) + c\Phi'(0)] - i\alpha\Phi(0)\}[U'(0)\varphi_3(0) + c\varphi_3'(0)] = \\ = -i\alpha\varphi_3(0)[U'(0)\Phi(0) + c\Phi'(0)] \quad (20)$$

Let us simplify equation (20). Let us introduce the following notations:

$$z = c\left(\frac{\alpha R}{[U'(0)]^2}\right)^{1/3}, \qquad u + iv = \left[1 + \frac{[U'(0)\Phi(0)]}{c\Phi'(0)}\right]^{-1}$$
$$\frac{U'(0)\varphi_3(0)}{c\varphi_3'(0)} = -F(z), \qquad F^*(z) = \frac{1}{1 - F(z)} \quad (21)$$

Here F(z) is a Tietjens function, tables of which are contained in [6]. Equation (20) when expressed in terms of the notations in (21) will, after uncomplicated transformations, appear in the form

$$F^*(z) = u + iv + \frac{U'(0)\, Y_{11}}{i\alpha} \tag{22}$$

The function Φ, in terms of which we can express u + iv, is determined by the solution of equation (15). Presenting this solution in the form of a series in terms of powers of α^2, and limiting ourselves to the principal terms, we obtain [6]

$$u + iv = cU'(0)\left[\frac{1}{\alpha(1-c)^2} + \omega\right], \quad \omega = \frac{1}{cU'(0)} + K(c)$$
$$K(c) = \int_0^1 \frac{dy}{(U-c)^2} = -\frac{1}{cU_c'} + \frac{U_c''\ln c}{(U_c')^3} - \frac{U_c''\pi i}{(U_c')^3} + \ldots \tag{23}$$

Let us substitute the value for Y_{11} from equation (13) in equation (22), and let us isolate the real and the imaginary parts. We find that

$$F_r^* = u - \frac{mU'(0)(c_0^2/c - c)}{d^2 + m^2\alpha^2(c_0^2/c - c)^2}$$
$$F_i^* = v - \frac{U'(0)\, d}{\alpha\left[d^2 + m^2\alpha^2(c_0^2/c - c)^2\right]} \tag{24}$$

where

F_r^* and F_i^* are the real and the imaginary parts of the function $F^*(z)$ respectively.

Let us note that the link between the pressure and the deformation sometimes is given in the form [3]

$$\eta = p' K_1 e^{i\theta}$$

that is, in accordance with equation (10), it is taken that

$$Y_{11} = i\alpha c K_1 e^{i\theta}$$

and not considered as a concrete form of a deformable surface. With equation (13) in mind, it is not difficult to see that for the model of a deformable surface adopted here

$$K_1 = \{[m\alpha^2(c_0^2 - c^2)]^2 + d^2c^2\alpha^2\}^{-1/2}$$
$$\operatorname{tg}\theta = -\frac{dc\alpha}{m\alpha^2(c^2 - c_0^2)}$$

that is, K_1 and θ depend on the physical parameters of the disturbance wave and on the parameters of the deformable surface.

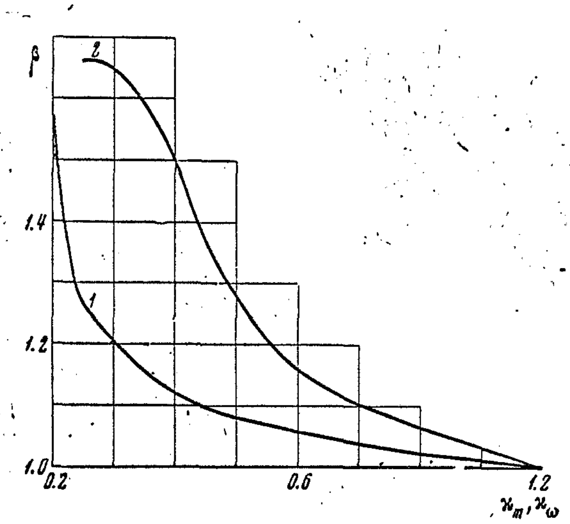

Figure 3

Based on the foregoing, the construction of the neutral stability curve for fixed parameters of a deformable surface can be carried out in the following sequence. Find F_r* and F_i* in the tables for each z. Solve equations (24) and (21) for α and c. Compute the corresponding R number by solving the corresponding equation at (21). Corresponding to the R number for loss of stability are $z = 3.21$, $F_i^* = 0.58$, and $F_r^* = 1.49$.

Figure 3 shows the results of the R number computations for loss of stability. Curve 1 depicts the dependence of $\beta = R/R_1$ on the parameter for the mass $X_m = k_m/k_{m1}$ when $C_{o_m} = 0.75$, $k_\omega = 4.56 \cdot 10^{-5}$, $d = 0.1$, $k_{m1} = 1{,}8 \cdot 10^4$. The R_1 number corresponds to the R number when $k_m = k_{m1}$, differing little from the R number for a rigid surface. Curve 2 is for the dependence of β on $\varkappa_\omega = k_\omega/k_{\omega 1}$ when $\varkappa_m = 0.4$, $c_{o_m} = 0.75$, $d = 0.1$, $k_{\omega 1} = 7.4 \cdot 10^{-4}$. It should be pointed out that the selection of the values for c_{o_m} and d in these computations was more or less arbitrary.

Submitted 13 May 1969.

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